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EFFECTS OF THE VARIATIONS OF RECOMBINATION COEFFICIENT AND SCALE HEIGHT ON THE STRUCTURES OF THE IONIZED REGIONS

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ABSTRACT. The paper considers in detail the effects of the height gradient of the recombination coefficeint and of scale height on the altitude-distribution of ionization for the ionospheric regions, D, E, F_1 and F_2. The characteristics of the gradients are estimated from experimental data. General equations are derived for calculations of height distributions of electrons, taking into account the gradients of recombination coefficient and scale height. (For deriving the formulae for the D region the results of electron and ion distributions as already calculated by the author (Mitra, 1951*b*) have been utilised. For the E region the calculated distribution shows that it is not markedly different from the Chapman type of distribution. For the F-region, the gradients as noted above are found to have profound effects. It is found that a simple Chapman layer (which may be identified with the F_1 region) 'bifurcates,' as it were, into two regions—F_1 and F_2, mainly as a result of the height gradient of the recombination coefficient. This lends support to the contemporary suggestion that the F_2 region is not formed by an ionization process distinct from that for the F_1 region. The effect of the scale height gradient is also estimated. While not affecting markedly the shape of the distribution curve, it makes significant contribution to the value of the ionization density. It is further found that the bifuracation effect is more pronounced under summer than under winter conditions. However, for a full explanation of the anomalous behaviour of the F_2 region one has to take into account the effect of tidal motions under the influence of the geomagnetic field.

I. INTRODUCTION

The assumption of a simple Chapman layer formed by absorption of monochromatic solar radiation in an isothermal atmosphere of constant composition has been of great help in the theoretical study of the characteristics of the ionized regions of the upper atmosphere. However, the assumption has got its limitations which make it unsuited for accurate predictions of the behaviours of the ionospheric regions. The limitations are due to the highly idealised nature of the assumptions made, namely, that (i) the absorbed radiation is monochromatic and (ii) that the temperature, the molecular weight (that is, the scale height H) and the recombination coefficient α remains constant with height. None of these assumptions holds strictly. In fact, there are regions of the ionosphere in which one or more of the above parameters diverge widely from the ideal assumption. Consider first the assumption regarding the monochromatic nature of the ionizing radiation. It is well known that the absorbed radiation is never monochromatic. However, one may reasonably assume that the band of wavelength which is absorbed for a particular process of ionization is narrow and as such may be approximately regarded as monochromatic. The assumptions regarding the constancy of temperature and recombination coefficient are, however, quite at variance with observed facts. Various indirect upper atmospheric observations, as also direct measurements by high-flying rockets show that there are large temperature gradients in the upper atmosphere at the heights where the ionospheric regions are formed. Again, the ionospheric region E is formed round the height where O_2 concentration changes rapidly with height due to dissociation. As such, calculations on the altitude distributions of ionization for the E layer is hardly justifiable without taking these factors into account. Further, it is known that the value of the recombination coefficient α depends strongly on temperature and pressure. It is thus imperative to take account of these factors in estimating the height-ionization distributions of the ionized regions.

Amongst the attempts made in recent years to calculate the height-ionization distributions by taking account of the variabilities of the parameters listed above the following may be mentioned. Mitra (1951*b*) has considered the variations of temperature and recombination coefficient with height for the D layer, Pfister (1950) for the E layer, and Bates and Massey (1946) and later Bates (1949) have considered the effect of a variable recombination coefficient on the F-layers. Mention may also be made of the work of Gledhill and Szendrei (1950) who considered mathematically the effect of a linear temperature gradient on an otherwise Chapman layer and of Nicolet (1950) who considered the effects of both recombination coefficient and temperature on a layer produced by a band of solar ultraviolet radiation. These works have materially advanced our concept of the ionospheric layers; but in most cases they are not satisfactorily complete. In the present paper an attempt has, therefore, been made, firstly, to derive general formulae for calculating the height distribution of ionization taking into account the possible variabilities of H (that is, temperature and molecular composition) and of α. Secondly, the available experimental data regarding the nature of variations of the above parameters with height are collected and scanned. Finally, with the help of the formulae and the collected data, the height-ionization distributions of the D, E, F_1 and F_2 regions are calculated.

2. BASIC FORMULAE

Before deriving the equations to be used in the calculation, it will be convenient to list the formation characteristics of the Chapman layer.

(*i*) *Chapman layer—H and α constant* (Chapman, 1931) *:* We consider an isothermal atmosphere traversed by monochromatic solar radiation. The particle concentration is given by the exponential law : $n = n_0 \exp(-h/H)$, where h is the height measured from a datum level, and H is the scale height. The rate of electron production is given by

$$q = An_0Q \exp(-n_0 AH_0 \sec\chi\, e^{-h/H}), \qquad \dots (1)$$

where A = absorption cross-section of the active atmospheric constituent,

$H_0 = \dfrac{kT}{mg}$ = scale height,

χ = solar zenith angle,

h = height measured from a datum level where the particle concentration is n_0. q has a maximum q_m given by

$$q_m = q_0 \cos\chi, \qquad \dots (2)$$

at the height h_m or z_m [$= (h_m - h_0)/H_0$] given respectively by

$$h_m = H \log n_0 A H_0 \sec \chi, \quad \ldots (3)$$

$$z_m = \log \sec \chi. \quad \ldots (4)$$

It is to be noted that the height (h_m) at which the rate of ion production is maximum is also the same at which the electron or ion concentration is maximum.

Another important quantity is $n\,(h_m)$ the particle concentration at the height of maximum ion production and is given by

$$n\,(h_m) = \cos \chi / (A H_0). \quad \ldots (5)$$

(*ii*) *H* (*scale height*) *varying with height :* We consider how the Chapman formulae will be modified if H varies linearly with altitude, i.e.,

$$H = H_0 + ah,$$

where H_0 is the value of the scale height at the datum level, and a is the gradient of scale height. For such an atmosphere the height variation of particle concentration n is no longer given by the exponential law, but by

$$n = n_0 (1 + Bh)^{-(1+1/a)}$$

where $$B = a/H_0$$

Eqns. (1), (2), (3) and (5) are now replaced by

$$q = A n_0 Q \, (1 + Bh)^{-(1+1/a)} \exp [-n_0 A H_0 \sec \chi \, (1 + Bh)^{-1/a}] \quad \ldots (6)$$

$$q_m = q_0 (\cos \chi)^{1+a} \quad \ldots (7)$$

$$h_m = H_0 \left[-\frac{1}{a} + \frac{1}{a} \frac{(A n_0 H_0 \sec. \chi)^a}{1+a} \right] \quad \ldots (8)$$

$$n(h_m) = \frac{\cos \chi \, (1+a)}{\ldots} \quad \ldots (9)$$

where H_m is the value of scale height at the height h_m.

(*iii*) α (*recombination coefficient*) *varying with height :* We next obtain formulae for the height of maximum ionization and the value of the maximum electron concentration, taking into account the variability of the recombination coefficient in addition to the variation of scale height just considered.

We note that unlike that in the Chapman layer, the height at which the rate of ion production is maximum is *not* the same as that at which the ionization density is maximum.

We have

$$\frac{dN}{dt} = q - \alpha N^2 \quad (10)$$

where N—number density of electrons/cm^3, α—coefficient of recombination.

For the regions where α is large $\frac{dN}{dt} = 0$, throughout the day, specially during noon. For the region where α is small, such as the F_2 region, $\frac{dN}{dt}$ is usually much different from zero. Sometime after noon, however, its value decreases to zero We thus have

$$N = (q/\alpha)^{\frac{1}{2}} \qquad \text{when } \frac{dN}{dt} = 0, \qquad \ldots (11)$$

$$N = \left\{ \left(q - \frac{dN}{dt} \right) / \alpha \right\}^{\frac{1}{2}} \text{when } \frac{dN}{dt} \neq 0. \qquad \ldots (11a)$$

As α depends on temperature, pressure and electron concentration (depending on the region concerned), one may put

$$\alpha = \alpha_0 f(N, p, T), \qquad \ldots (12)$$

where α_0 is a constant, and f is the function giving the variation of α with one or more of the parameters : temperature, height and electron concentration.

We thus have combining Eq. (6) and (11, 11a) with Eq. (12),

$$N^2 = \frac{A n_0 Q}{\alpha_0} \left[(1+Bh)^{(1+a)} \exp\left\{ -\frac{n_0 A H_0 \sec\chi (1+Bh)^{a}}{f} \right\} \right] \text{for } \frac{dN}{dt} = 0, \quad (13a)$$

$$N^2 = \frac{A n_0 Q}{\alpha_0} \left[(1+Bh)^{-(1+a)} \exp\left\{ -\frac{n_0 A H_0 \sec\chi (1+Bh)^{a}}{f} - \frac{dN}{dt} \right\} \right] \text{for } \frac{dN}{dt} \neq 0 \quad (13b)$$

3. SCALE HEIGHT AND RECOMBINATION COEFFICIENT VALUES

(*a*) *Scale height :* The scale height (that is, the temperature) of the atmosphere *below* 100 Km. is known with satisfactory precision from studies on abnormal sound propagation phenomena and from the luminosity of meteor trails as also from balloons and *V*-2 rocket flights. A standard temperature distribution, based on the above findings, has been tentatively assumed by the National Advisory Committee for Aeronautics (NACA) (figure 1). The distribution, of course, varies with the hour of the day, the season of the year and perhaps also with the solar cycle. Nevertheless, it may be expected that there will not be large deviations from the general shape of the NACA standard variation.

The scale height *above* 100 Km. is, however, not known with such precision. Evidence from auroral, ionospheric and other measurements all point to the existence of a rising scale height above 100 Km., reaching a value which may be as high as 70 Km. in the F_2-region heights.

Scale height in region E : The most accurate measurements of scale height for this region made so far are by Pfister (1950) who took account of the effect of the terrestrial magnetic field. The values obtained by him for Washington range from 5.5 to 12 Km. Pfister's report does not specifically state the amount of seasonal variation in H, but a ratio of about 1.5 or more between summer and winter values is likely.

The above values of H are obtained on the assumption of an isothermal atmosphere; we may, therefore, consider them as representing the actual values at the base of the layer (H_0), say at 100 Km. height. The scale height at any altitude

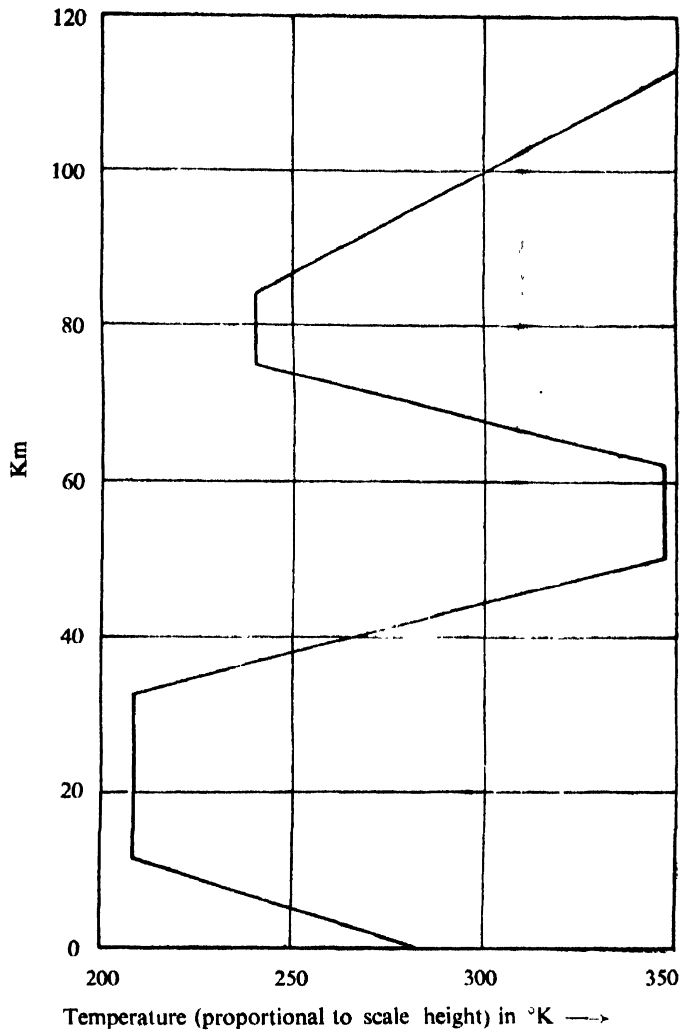

FIG. 1

Idealised temperature distribution (i.e., the scale height distribution) adopted by the National Advisory Committee for Aeronautics (NACA), U.S.A.

higher than this may be obtained if the scale height gradient a is known. a can be estimated from the durinal variation curve for critical frequency. Thus, for an atmosphere in which the scale height increases linearly with height, the variation of critical frequency is given by (see sec. 2)

$$f = f_o (\cos \chi)^{(1+a)/4} \qquad \ldots \quad (14)$$

In Table I are given (i) the values of the exponent of cos χ as obtained from routine ionospheric data of Washington, Kochel, Huancayo, Watheroo (Harnischmacher, 1951) and Calcutta, and (ii) the values of the scale height gradient obtained by the help of Eq. (14).

It is obvioùs from the table that the scale height gradient is a function of geographical latitude. But values round 0.30 in summer and 0.15 in winter may be

TABLE I

Location	Latitude	Value of exponent $(1+a)/4$		Value of a	
		Summer	Winter	Summer	Winter
Kochel	47·7 N	0.350	0.270	0.40	0.10
Washington	38.7°N	0.320	0.270	0.30	0.10
Watheroo	30.3°S	0.310	0.260	0.24	0.04
Calcutta	22.5 N	0.300	0.265	0.20	0.05
Huancayo	12.0 S	0.350	0.310	0.40	0.24

taken as representative values. The value of H at any height below region F_1 can be easily estimated now.

Region F_1 : Nicolet (1947) has estimated the scale height of this region as 30 Km. Bates and Massey (1946) also favour a value of the same order. Values higher than this have also been reported. Thus, Appleton and Beynon (1947) have obtained a value as high as 55 Km, while Kellog (1950) reports a value 43 Km. for equatorial latitudes. No report regarding the seasonal variation of the quantity is available, but, a summer to winter ratio as calculated from E-region-ionospheric parameters seems not unlikely.

For evaluation of the scale height gradient for this region the same method, as used for region E, can be utilised. The value obtained for middle latitude is found to be about 0.3.

Region F_2 : Estimation of scale height for the F_2 region has been made by various authors either on the basis of a parabolic layer or of higher order approximations to the Chapman type of distribution. The values quoted range from one as low as 20 Km. (Pekeris, 1940) to one as high as 186 Km. (Grace, Kelso and Miller, 1949). The normal range of the scale height is, however, from 40 to 70 Km. (Appleton, 1939; Baral and Mitra, 1950; Gerson, 1951), with a seasonal variation from about 70 Km. in summer to about 50 Km. in winter. It is clear that the scale height for this region depends on various factors, such as solar zenith angle, latitude of the place and the atmospheric height concerned.

It is important to remember that the above values of scale height are only the higher limits of the same. This is because in all these measurements, variations of recombination coefficient and temperature with height have not been taken into account. The effects of both these gradients will be to increase the thickness of the layer, so that the values cited above are too high. The ratio of winter to summer temperatures, however, will not be so much in error as the absolute values of either of these quantities.

The gradient a of scale height for this region cannot be determined as above. It is, however, possible that the gradient for this region is the same as that for region E. We, therefore, assume $a = 0.3$.

(*b*) *Recombination coefficient :* It has been shown by Bates and Massey (1946) that the value of the effective recombination coefficient may be expressed as

$$\alpha = \alpha_e + \lambda\alpha_i + \frac{1}{NT}\frac{dT}{dt} + \frac{1}{N(1+\lambda)}\frac{d\lambda}{dt} \qquad \dots (15)$$

with

$$\lambda = \beta N_n/(KN_n + \rho N + \alpha_i N^2 + q - \alpha_e N^2) \qquad \dots (16)$$

where

α_e —electronic recombination coefficient,
λ —negative ion to electron ratio,
β —coefficient of attachment,
K —coefficient of collisional detachment,
ρ —coefficient of photo-detachment,
α_i —coefficient of mutual neutralisation.

(*i*) *Region D :* For the D region the important operative processes are : attachment, photo-detachment, mutual neutralisation and collisional detachment. On account of the relatively high gas densities involved, it is probable that electrons become attached to oxygen molecules mainly through the Bloch-Bradbury process. The coefficient β associated with this is given by $\beta = 10^{-14} + 1.5 \times 10^{-12}\,p$ (p—pressure in mm.). For mutual neutralisation, a value 10^{-8}cm^3/sec. may be assumed (Bates and Massey. 1946). For the coefficient of collisional detachment a reasonable value is 5×10^{-16} cm^3/sec. The photo-detachment rate is assumed to be 0.35/negative ion/sec.

It is at once seen that α, under such conditions, will be given by (see also Mitra, 1951 *b*)

$$\left.\begin{aligned} \alpha &= \beta n\alpha_i/(Kn+\rho) \\ &= \alpha_i\,[n\,(10^{-14} + 1.5 \times 10^{-12}p)]/(Kn+\rho) \\ &= \alpha_i\,\lambda\,(h) \end{aligned}\right\} \qquad \dots (17)$$

$\lambda\,(h)$ as given by the author in a previous paper (1951 *b*) is illustrated in figure 2.

(*ii*) *Region E :* Until recently it was believed that the recombination coefficient for the E region is constant (Appleton, Naismith and Ingram, 1937; Bates and Massey, 1946). Recent measurements by Baral and Mitra (1950) have, however, shown that the coefficient for this region is variable, though by a small amount. This view has also been confirmed by other workers (Weekes, 1950; Pfister, 1950; Mcleish, 1948). The coefficient is, in general, smaller at day-time than at night-time. Further, for a chosen hour, the summer values are, in general smaller than the winter ones. It is possible that these variations are due to variations in atmospheric temperature at the heights concerned. This is because the temperature at E region height is known to be greater in summer than in winter and greater in day than at night. A quantitative expression for this relation may also be derived by application of Eqns. (15,16) (Baral and Mitra, 1950). It is known that β for

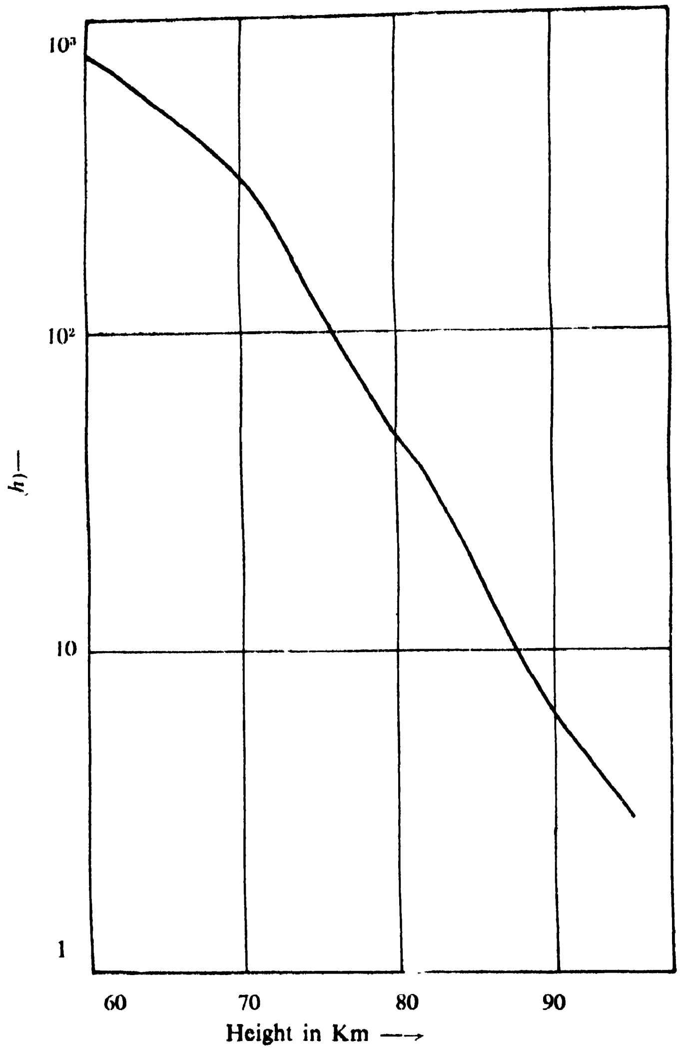

Fig. 2

The ion/electron number density ratio, λ, drawn as a function of height for the D-region. (After Mitra, 1951*b*.) The recombination coefficient α, at any height is proportional to λ.

this region, is temperature-dependent, being 6×10^{-14} cm³/sec. at 250° K and 1.5×10^{-14} cm³/sec. at 1000° K, so that one may assume

$$\beta = \eta / T,$$

where $$\eta = 1.5 \times 10^{-11} \text{ degree cm}^3/\text{sec.}$$

This gives from Eqn. (15),

$$\alpha_E = \eta \, \alpha_i / KT = \alpha_i (\eta / K) \, T^{-1}$$

In view of the uncertainties in the value of the exponent of T, one may write, in general,

$$\alpha_E = \alpha_i (\eta / K) \, T^{-r} \qquad \ldots (18)$$

Also since $T_j / T_d = H_j / H_d$ (j - Northern solstice months and d - Southern solstice months) Eq. (18) may be written as

$$\frac{\alpha_j}{\alpha_d} = \left(\frac{H_d}{H_j}\right)^r \qquad \ldots \quad (18a)$$

The variation of (H_j/H_d) for different values of r and for $\alpha_d/\alpha_j = 1.5$ (as experimentally observed) is shown in Table II.

TABLF II

r	1	1.5	2.0
H_j/H_d	1.50	1.30	1.23

The actual variations of H_j/H_d observed is nearabout 1.5-2.0, so that r is in the neighbourhood of 1. Hence we can write

$$\alpha_E = \alpha_o (1 + Bh)^{-1} \qquad \ldots \quad (18b)$$

(*ii*) *Region* F_2 : The variation in the value of α in the F_1 region is similar to that in the E region. An expression similar to that for the E region may, therefore, be assumed for the recombination coefficient :

$$\alpha_{F_1} \quad \alpha_o (1 + Bh)^{-1} \qquad \ldots \quad (19)$$

(*iv*) *Region* F_2 : The recombination coefficients for this region are usually measured without taking account of tidal phenomena. Such measurements made at Calcutta (Baral and Mitra, 1950) showed large variations of the coefficient with height, with electron concentration and with the solar zenith angle. In view of the approximate nature of these values (due to the neglect of the tidal terms) new measurements have recently been made by Mitra (1951*a*) in which the tidal phenomena have been taken into account. The results obtained by him show that the values of the coefficient, even when tidal terms are considered, are subject to variations. The nature of these variations is similar to that obtained above. The values of α are, however, lowered, as also the amplitude of the variations; the summer value being 1.5×10^{-11} cm^3/sec, and the winter value 3×10^{-11} cm^3/sec.

It is now necessary to have a quantitative expression for α relating it to the different variable parameters. This can be obtained by utilising Eq. (15, 16). We readily get, when proper approximations are made,

$$\alpha_{F2} = \eta n/TN = \frac{\eta n_o}{T_o N_e}(1 + Bh)^{-\left(2+\frac{1}{a}\right)}$$

$$= \frac{\gamma_c}{N_-}(1 + Bh)^{-\left(2+\frac{1}{a}\right)} \qquad (20)$$

It is probable that this equation is in error by a large amount; in particular the exponent of $(1 + Bh)$ appears to be much too large. For instance, with $\frac{1}{a} = 3$, the factor $(1+Bh)^{-\left(2+\frac{1}{a}\right)}$ decreases to more than $\frac{1}{10}$th of its value at 100 Km.

We therefore write

$$\alpha_{1/2} = \frac{\gamma_0}{N_e}(1+Bh)^{-g} \qquad \ldots \quad (20a),$$

where the value of g is in the neighbourhood of 5 in order to make the variations in $\gamma_0\ (1+Bh)^{-g}$ conform to experimental observations.

4. STRUCTURE OF THE D REGION

It is now generally accepted that D region is produced by ionization of O_2 at the first ionization potential (12.2 eV) as first suggested by Mitra, Bhar and Ghosh (1938)*. Calculations, based on this hypothesis, of the values of the electron concentration and of the scale height at various altitudes in this region have recently been made by the present author (Mitra, 1951*b*). The very close agreement between the calculated and the experimental values (Table III) provides convincing proof of the correctness of the hypothesis.

TABLE III

Electron concentrations (cm^{-3}) at				Scale height (Km) for					
60 Km		90 Km		16 Kc/s		43 Kc/s		113 Kc/s	
Exptl.	Theor.	Exptl.	Theor.	Exptl.	Theor.	Exptl.	Theor.	Exptl.	Theor
2.5×10^2	2.6×10^2	1.5×10^4	2×10^4	$5.5 \pm .1$	5.5	$4.8 \pm .1$	4.6	$2.8 \pm .1$	2.6

It is to be noted that calculations for the height-ionization distribution are rendered difficult and laborious by the fact that the scale height $H(= kT/mg)$ and the recombination coefficient (α) are not only *not* constant with height, but vary non-uniformly. However, one may assume that the D-ionization is comprised within the heights 30-100 Km. and divide up the region into five separate parts (Table IV) as per figure 1. The calculations may then be carried out separately for each of these five parts. This has been done by the author (and, with the assumption that the effective ionization process is photo-ionization of O_2 at its first ionization potential) and the height-ionization distribution of electrons and ions for the entire D region obtained. The results are depicted in figures 3 and 4. The following points may be noted in the characteristics of the curves in this figure.

(i) As the ion-electron number density ratio is very high (about 10^3 times the number of electrons at a height of 60 Km.), the ions, notwithstanding their

*The photo-ionization of sodium (Jouast and Vassy, 1941; Vassy and Vassy, 1942) is also sometimes suggested. It has a very low ionization rate, and, as will be shown by the present author in a later paper, is more likely to cause the irregularly occuring region called sporadic D rather than the regular D layer.

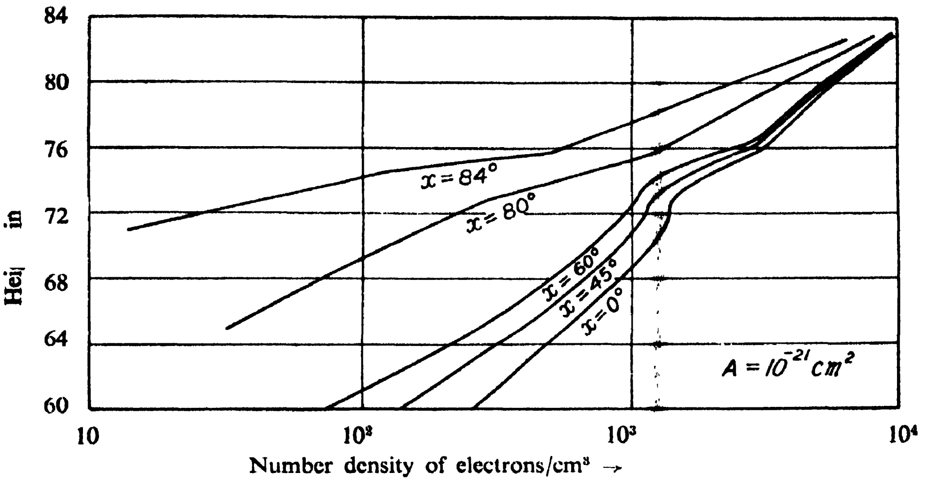

FIG. 3

Altitude distribution of electron concentration for the D region for $A = 10^{-21}$ cm² on the assumption that the effective ionization process is the photo-ionization of O_2 at the first ionization potential. The variation with solar zenith angle, χ, is shown by drawing five separate distributions for five different values of χ, namely $\chi = 0°, 45°, 60°, 80°$, and $84°$.

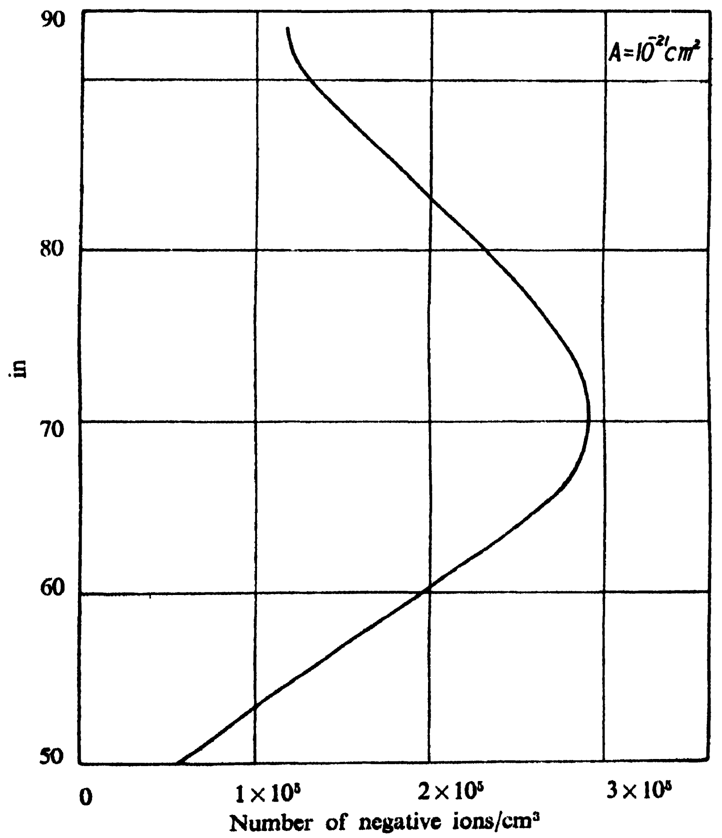

FIG. 4

Altitude distribution of negative ion concentration for the D region at the first ionization potential of O_2 for $A = 10^{-21}$ cm².

low mobility, will play the dominating role (in contrast to that in the higher ionospheric regions) in radio wave propagation specially of long and very-long waves.

TABLE IV

Region	Range (Km)	Temperature (T_0) at reference level (°K)	Temperature gradient (γ) (°K/Km)	n_0 (cm^{-3})
I	32-50	220°	9.2	5×10^{16}
II	50-63	350°	0.0	4×10^{15}
III	63-76	350°	−10.4	1.5×10^{15}
IV	76-83	240°	0.0	3.5×10^{14}
V	83-100	240°	4.5	1.4×10^{14}

(ii) The shape of the height-ionization distribution curves (specially for electrons) is totally different from that of the Chapman distribution. While the distribution for ions still maintains some similarity with the Chapman distribution, that for electrons is entirely different. The latter has no well-defined maximum but rises monotonously to a value of about 10^4/cm^3, and merges with the tail of the E region. (If a very small value for A is assumed, then there is a slight dip in the distribution curve just below the E region.)

(iii) The changes in the distribution curve with the variation of χ (solar zenith angle) is significant. With the decrease of χ, the distribution curve as a whole moves up and undergoes changes in shape. A special feature, when the value assumed for A is very low, is the sudden increase of the ionization gradient from 73 to 76 Km. This occurs for all values of χ, the gradient being sharpest for $\chi = 0°$, and decreasing with the increase in the value of χ.

It is possible to represent the above characteristics with simple equations, suitable for numerical calculations, if, as a first approximation, the small irregularities in the curves are ignored.

Taking first the case of electron distribution, we note from inspection of figures 3 and 4 that the distribution curve for a given value of χ is composed of four distinct sections, each section being exponential in shape. These sections are : (1) 60* to 71 Km., (2) 71 to 73 Km., (3) 73 to 76 Km., and (4) above 76 Km.

Now, let N_r represent the value of the electron concentration at any height h_r in the section r, and N_{ro} be the value of the same at the lower limit of the section concerned. Then for any of these sections we can write

$$N_r = N_{ro} (\cos \chi)^p \exp [a_{ro} (\sec \chi)^{nr} (h_r - x_r) \qquad \ldots (21)$$

where $r = 1, 2, 3, 4$ and $x_1 = 60$ Km., $x_2 = 71$ Km, $x_3 = 73$ Km. and $x_4 = 74$ Km.

*The base of the layer may lie even below 60 Km., specially for lower values of A; but, for the present purpose we may neglect such ionizations at the very low levels.

The values of the parameters involved in the equations are given in Table V.

TABLE V

Region r	Range in Km.	p	a_r	n_r
1	60-71 (h_1)	2	0.15	0.60
2	71-73 (h_2)	0	0.04	2.00
3	73-76 (h_3)	0	0.28	0.28
4	76- (h_4)	0	0.18	0.20

For the distribution of ions more exact expressions are possible, specially for the lower heights 60 to 80 Km. which are of importance to long wave propagation. Since the negative ion to electron number density ratio is λ, the concentration of negative ions is given by

$$\begin{aligned} N_i &= \lambda\sqrt{q}/[\alpha(1+\lambda)] \\ &= \lambda\sqrt{q}/[\alpha_i(1+\lambda)\lambda]. \end{aligned} \qquad \ldots (22)$$

At the lower heights (60-80 Km.), $\lambda >> 1$. Hence

$$N_i = \left(\frac{q}{\alpha_i}\right)^{\frac{1}{2}}, \qquad (23)$$

which is a very convenient expression.

The ionic distribution is thus not affected by variations of recombination coefficient, and is only sensitive to variations in temperature.

5. STRUCTURE OF THE E REGION

According to current ideas the E region is formed by ionization of O_2 round the height of dissociation transition $O_2 \rightarrow O + O$. A strong point in favour of this hypothesis is that the measured heights of the E-peak ($\simeq$ 100 Km.) lie close to the calculated height of the transition region. However, there has been some difference of opinion regarding the exact location of the E region with reference to the transition region.

According to Penndorf (1949) the E region is formed *above* the transition level. But recent work of Moses and Ta-You Wu (1951) seems to indicate that it may be *inside* the transition level. In the first case the concentration of O_2 molecules drops with height more slowly than in the second case. However, we may write generally for the concentration (n) of the active particles (O_2-molecules) at any height within the dissociation transition range as

$$n = n_0 (1 + Bh)^{-\left(1+\frac{b}{a}\right)},$$

where the new term takes account of the effect of the additional drop in the number-density because of dissociation. Its value may be greater or less than 1.

For the region under consideration α is a function only of temperature, and is given by $(T/T_0)^{-r}$ where r is approximately equal to 1. Now T, is a function of height, so that, assuming the linear rise of T we write,

$$T = T_0(1+Bh).$$

Hence, combining Eqn. (13) of sec. (2) with the above relation, we have

$$N^2 = \frac{A n_0 Q}{\alpha_0}(1+Bh)^{r-\left(1+\frac{b}{a}\right)}.\exp\left\{\frac{-n_0 A H_0 \sec\chi}{r-(1+{}^{b}/_{a})}(1+Bh)^{-\frac{b}{a}}\right\}$$

$$= N_0^2(1+P)^{r-\left(1+\frac{b}{a}\right)}\exp\left[\left\{1+\frac{a}{b}(1-r)\right\}\left\{1-(1+P)^{-b/a}\sec\chi\right\}\right] \tag{24}$$

where we have put $\qquad P = \dfrac{B(h-h_{m0})}{1+Bh_{m0}},$

h_{m0} being the height at which the electron concentration is maximum, and is given by

$$(1+Bh_{m0})^{-\frac{b}{a}} = [1+\tfrac{a}{b}(1-r)]/(n_0 A H_0 \sec\chi).$$

But as $r = 1$, Eq. (24) may be written as

$$\frac{N}{N_0} = (1+P)^{-\frac{b}{2a}}\exp\tfrac{1}{2}\left[1-(1+P)^{-\frac{b}{a}}\sec\chi\right] \tag{25}$$

The equation is further simplified if account is taken of the fact that for E region conditions $a(h-h_{m0}) << H_0$. Hence,

$$\frac{N}{N_0} = \left[1-\frac{bz}{2R}\right]\exp\tfrac{1}{2}\left[1-\left(1-\frac{bz}{R}\right)\sec\chi\right], \quad \ldots \tag{25a}$$

where $\qquad z = (h-h_{m0})/H$ and $R = (1+Bh_{m0})$.

Figure 5 gives the distributions of electron concentration for the E region as deduced from Eq. (25*a*) for different values of b.

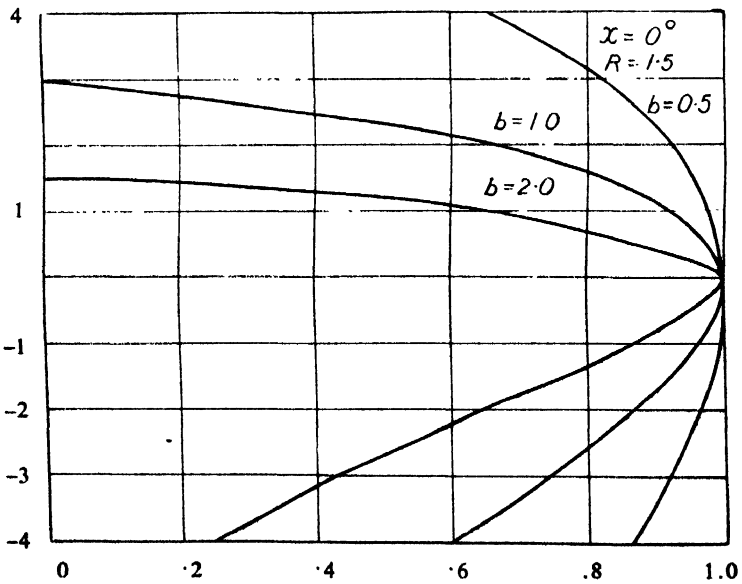

Fig. 5. Altitude distribution of electron concentration for the E-region for $\chi = 0$ and $R = 1.5$.

6. STRUCTURES OF REGIONS F_1 AND F_2

According to earlier views, production of F_1 and F_2 regions was ascribed to photo-ionizations of two different atmospheric constituents at two different levels. For examples, Bhar(1938) postulated that F_1 region is produced by ionization of N_2 and F_2 region by ionization of O. According to contemporary ideas (Bradbury, 1938; Bates 1949), however, the F_1 and F_2 regions form together, even in daytime, one single region produced by ionization of *only one* atmospheric constituent (atomic oxygen). This single region is "bifurcated," as it were, by the effect of the rapid decrease of the recombination coefficient with height, and thus appears as two separate regions of ionization. The quantitative analysis of the effects of decrease of recombination coefficient and of increase of scale height with height, and of the height distribution of electrons in the composite F_1 and F_2 regions that is to follow will show that the above hypothesis is fully justified. It is to be noted, however, that according to some authors (Martyn, 1948), the effect of tidal forces on the movement of electrons and ions in these regions (under the influence of the geomagnetic field) will also contribute to the bifurcation effects. This aspect of the problem has also been studied by the author of this note. Preliminary analyses show that the bifurcation effect, though present, is not large. The subject will be discussed fully in a separate communication.

We shall now, first consider the effect of the recombination coefficient decreasing with height regarding the scale height to remain constant. The more general case, where both the recombination coefficient and the scale height vary with altitude, will be considered next. This procedure will enable one to assess properly the relative contributions of the two variations to the bifurcating process.

(*a*) *Height distribution of electrons in the composite F-region when the recombination coefficient* (α) *decreases with altitude* (*scale height H is constant*).

For the isothermal atmosphere under consideration α is sensibly constant in the F_1 region. But, from a height z_0 (which may be regarded as the boundary between the F_1 and the F_2 regions), above h_0, the recombination coefficient decreases rapidly with height. The actual value of z_0 is a matter of guesswork. It cannot be very small (compared to the scale height)—for otherwise the F_1-peak would rise above z_0 in winter months, and would consequently diverge widely from Chapman laws. This does not happen. The value of z_0 cannot be very large either, for we know that the F_2-region conditions begin from only a little distance above h_0. The height z_0 may thus be taken to be round 30 Km. ($\simeq H_0/2$).

We first consider the case of the steady state, that is, when $\frac{dN}{dt} = 0$. This is a highly idealized state, and exists only at height z_0 and even then round noon-time only. Calculations of the altitude-distribution of the electrons (N/N_0) will be carried out separately for the two regions, below z_0 and above z_0, because, as already emphasised, the nature and magnitude of the recombination coefficient gradient are different for the two regions.

(i) $(N/N_0)_s$ *for heights below* z_0 : $(N/N_0)_s$ for heights below z_0 is given by Chapman's equation (Sec. 2), namely,

$$\left(\frac{N}{N_o}\right)_s = \exp \tfrac{1}{2}(1 - z - e^{-z} \sec \chi),$$

the height of maximum ionization being given by

$$z_m = \log \sec \chi$$

(The subscript s in the ratio of ionizations denotes the value for the steady state.)

(ii) $(N/N_o)_s$ *for heights above* z_0 : For heights above z_o one may write

$$\nu = \gamma / N_o$$

where ν is a function of height. Let $\nu = \nu_o f(z)$.

Then $$\left(\frac{N}{N_o}\right)_s = \frac{q_o}{\nu_o} f^{-1}(z). \exp [1 - z - e^{-z} \sec \chi]. \quad ...(26)$$

The maximum electron concentration is

$$N_{mF_2} = \frac{q_o}{\gamma(z_{mF_2})} \exp [-z_{mF_2} - f'(z_{mF_2})/f(z_{mF_2})]. \quad .. \ (27)$$

At the height z_m given by

$$e^{-z_{mF_2}} \sec \chi - 1 = f'(z_{mF_2})/f(z_{mF_2}). \quad ... \ (28)$$

Since H is constant, $f(z)$ may be written as $f(z) = \exp [-p(z - z_o)]$ (see Sec.3). Further, ν_0 and α are related by the equation

$$\alpha = \nu_o / N_{zo}$$

We now have the following equations for the composite F-region consisting of F_1 and F_2 region :

$$\left.\begin{aligned} \left(\frac{N}{N_o}\right)_s &= \exp \tfrac{1}{2}(1 - z - e^{-z} \sec \chi) \text{ for } z < z_o \\ \left(\frac{N}{N_o}\right)_s &= \left(\frac{N_o}{Nz_o}\right) e^{-pz_o} \exp [1 + (p-1)z - e^{-z} \sec \chi] \\ &\qquad \text{for } z > z_o \end{aligned}\right\} \quad (29)$$

In figure 6 the distributions of electrons, for a given ionization process, as calculated from the above two equations are plotted. Two values of p are chosen : $p = 0.9$ and $p = 1.5$. For each of these values two separate curves are drawn representing the distributions for summer time and winter time conditions (i.e., for $\chi = 0°$ and $\chi = 45°$ respectively).

Even in this simple case, several interesting deviations from the Chapman distribution may be noticed. These are as follows :

(1) For values of p less than 1 the ionized region splits up at the top giving rise to a second maximum above the normal one. The lower normal maximum occurs, for the isothermal case considered, at exactly the same height as that of the maximum ion production rate, and is evidently the F_1-peak. The upper maximum, situated at a distance of $z_{mF_2} = 2.3$ for $\chi = 0°$ and of $z_{mF_2} = 2.6$ for $\chi = 45°$ is the peak of F_2 region. A single ionization process has thus given rise to both the F_1 and F_2 regions. The splitting up of the region at its top portion we may call *bifurcation*.

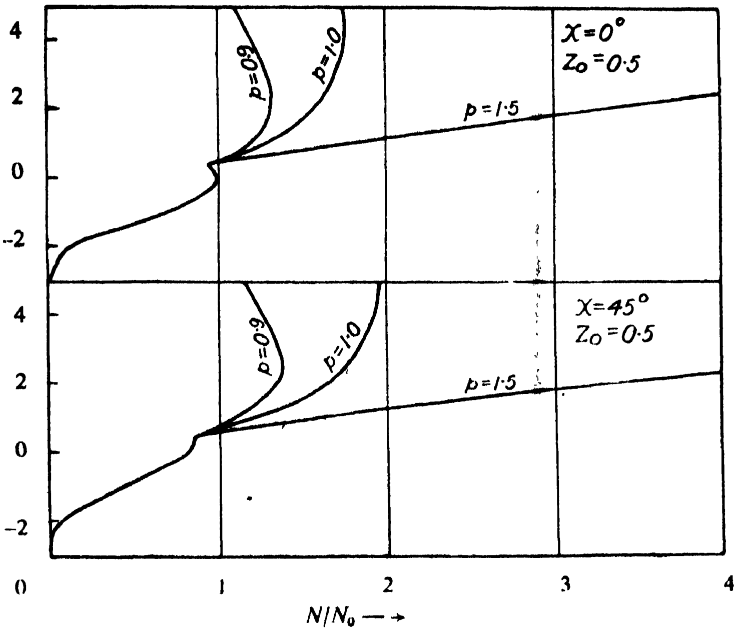

FIG. 6

Altitude distribution of electron concentration in the F-region on the assumption that dN/dt = 0, when the recombination coefficient decreases exponentially with height and scale height is constant. Both summer and winter time distributions are given, and z_0 is assumed to be 0.5 in both seasons. For each season three different gradients of recombination coefficient (*viz.*, p = 0.9, p = 1.0 and p = 1.5) are chosen.

(2) For values of p equal to or greater than 1, no well-defined second maxim exists. The electron concentration for such values of p decreases only slightly above the F_1-peak, and then increases monotonously upwards.

(3) The electron concentration at the second maximum (for the values of p for which it exists) increases as the value of p increases. For p equal to 0.9, the ratio $(N_{m}F_2/N_{m}F_2)$ is 1.34 in summer and 1.39 in winter.

(4) The height of the second maximum is given by

$$Z_{m}F_2 = \log_e [\sec \chi/(1-p)] \qquad \ldots (30)$$

This gives for p = 0.9, $z_{m}F_2$ = 2.3 and 2.6, for $\chi = 0°$ and $\chi = 45°$ respectively. Remembering that the value of the scale height is 70 Km. in summer and 50 Km. in winter, we obtain

$$(h_{mF_2} - h_0)_{\chi = 0^\circ} = 160 \text{ Km.}$$

$$(h_{mF_2} - h_0)_{\chi = 45^\circ} = 130 \text{ Km.}$$

The F_1-peak is exactly at h_0 for $\chi = 0°$ and 17 Km. above h_0 for $\chi = 45°$, so that the separation between the F_1 maximum and the second, that is the F_2 maximum is 160 Km. for $\chi = 0°$ and 113 Km. for $\chi = 45°$.

The above calculations have all been made on the assumption of a steady state i.e. $dN/dt = 0$. Such an assumption is not strictly valid for the regions where the recombination coefficeint is small, e.g. near the F_2 region. Since dN/dt has the dimension of q and is subtracted from it, the actual value of(N/N_0) at heights where $dN/dt \neq 0$ even during noontime, will be less than that obtained for the steady state.

It is thus generally seen that there will be well-defined upper maximum even for $p > 1$, contrary to the monotonous increase depicted in figure 6 for such cases.

Computation of the ionization distribution from Eq. (10) is necessarily elaborate and laborious. Such elaborate computations are not merited at this stage. We therefore use, instead, a very simple, albeit rough, method of evaluating (N/N_0).

We have, combining Eqns. (10) and (29)

$$\frac{N}{N_0} = \left(\frac{N}{N_0}\right)_s - \left[\left(\frac{dN}{dt}\right)/\gamma_0 N_0\right] \exp\left[p(z - z_0)\right] \qquad \dots (31)$$

dN/dt will evidently be a function of height. If (N/N_0) were given by Eqn. (29) then, roughly,

$$\frac{dN}{dt} = lN,$$

where l is a constant. Eqn. (31) then becomes

$$N/N_0 = C.\,(N/N_0)_s$$

where C is the correction term, being given by

$$C = [1 + l/\gamma_0.\, e^{p(z - z_0)}]^{-1}$$

For noon-time conditions the correction term C becomes unity at height z_0 and below i.e. in the F_1 region. This is possible only if (l/γ_0) is very small say, of the order 10^{-2}.

The distribution curves of (N/N_0), thus corrected by introduction of the factor C will represent more closely the actual distribution than only the (N/N_0) distribution. Figures 7 and 8 are drawn by assuming three different small values of l/γ_0 viz., $l/\gamma_0 = 0.05$, 0.03 and 0.01. It will be noticed that a well-defined maximum now exists even for $p > 1$.

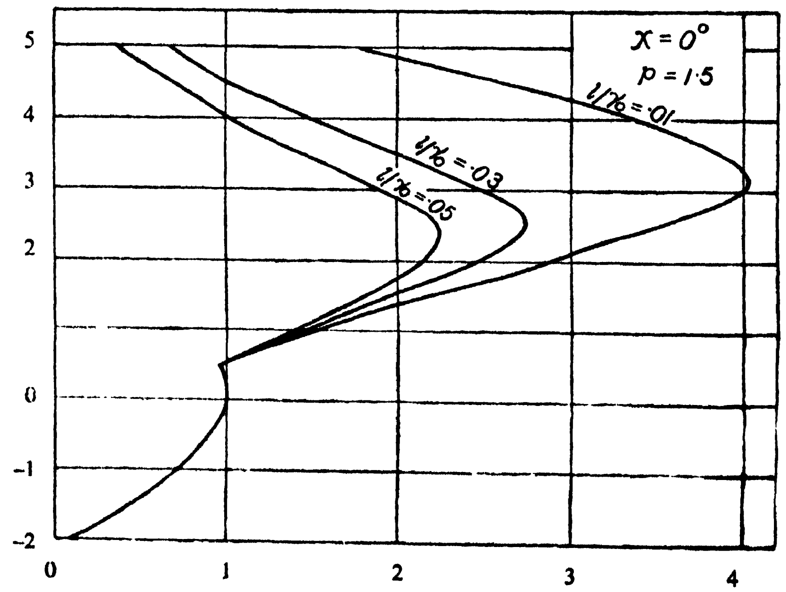

FIG. 7

Approximate distributions of electron concentration for the F-regions for $\chi = 0°$ and $p = 1.5$ and $dN/dt \neq 0$, and for three different values of (l/γ_0), *viz.*, .05, .03 and .01. Compare these curves with the cruve for $\chi = 45°$ in Fig. 8.

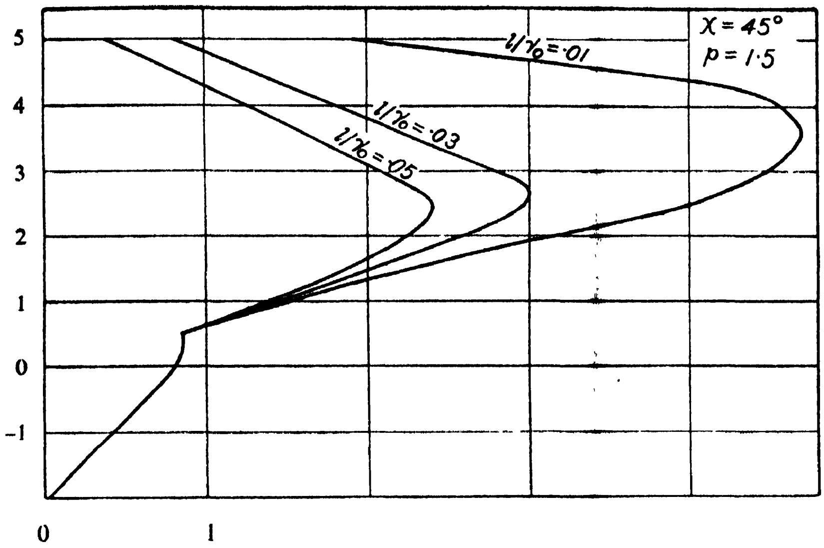

FIG. 8

Approximate distributions of electron concentration for the F-regions for $\chi = 45°$ and $p = 1.5$ and $dN/dt \neq 0$, and for three different values of (l/γ_0), *viz.*, .05, .03 and .01.

(*b*) *Height distribution of electrons in the composite F-region when the recombination coefficient* (α) *decreases and the scale height* (H) *increases with altitude.*

We now consider the more general case in which the recombination coefficient decreases with height according to Eqns. (19) and (20), and the scale height increases with altitude with a gradient of 0.3.

Let the boundary between the F_1 and the F_2 regions lie at a height b Km. (corresponding to the previously assumed value z_0) above h_{m0}. In contradistinction to the isothermal case, the recombination coefficient is not constant below b, but varies slowly with height—so slowly in fact that within a height of 50 Km. the recombination coefficient falls only to 2/3 of its value. Above b, however, the coefficient decreases very rapidly. If we assume, as before, that $b \simeq 30$ Km., then $P_b \simeq 0.1$.

We first consider the steady state, namely, when $dN/dt = 0$.

(i) $(N/N_0)_s$ *for heights* $h < b$ *:* The electron concentration for such heights is easily shown to be

$$\left(\frac{N}{N_0}\right)_s = (1+P)^{-\frac{1}{2a}} \exp. \tfrac{1}{2} \left[1-(1+P)^{1/a} \sec \chi\right] \qquad \ldots \quad (32)$$

(ii) $(N/N_0)_s$ *for heights* $h > b$ *:* As indicated in Sec. 3, the recombination coefficient for this region may be written as

$$\alpha = \frac{\gamma_0}{N_b}(1+Bh)^{-g}$$

Then, for the same ionization process as is operative for heights $h < b$, we have

$$N = \frac{An_0Q}{\gamma_0}(1+Bh)^{-(1+\frac{1}{a})} \exp\left[-(1+P)^{-1/a} \sec \chi\right].$$

Remembering that at $h = b$,

$$\alpha_0(1+Bb)^{-1} = \frac{\gamma_0}{N_b}(1+Bh)^{-g},$$

we have

$$\left(\frac{N}{N_0}\right)_s = \left(\frac{N_0}{N_b}\right)\left(\frac{1+Bb}{1+Bh_{mo}}\right)^{1-g}(1+P)^{g-(1+1/a)} \tag{33}$$

$$\times \exp\left[1-(1+P)^{\frac{l}{a}}\sec\chi\right]$$

At $P = P_b$, this equation should give the same value for $(N/N_0)_s$ as that given by Eqn. (32). Thus

$$\left(\frac{1+Bh}{1+Bh_{mo}}\right)^{1-g} = \left(\frac{N_b}{N_0}\right)^2 (1+P_b)^{1+\frac{l}{a}-g}\exp\left[(1+P)^{-l/a}\sec\chi-1\right] = F(b).$$

Hence, $$\left(\frac{N}{N_0}\right)_s = \frac{N_0}{N_b}F(b).\,(1+P)^{g-(1+\frac{l}{a})}\exp\left[1-(1+P)^{-l/a}\sec\chi\right] \tag{34}$$

In figure 9 are given the height distributions of $(N/N_0)_s$ for $b= 0.1$ and $g = 3.75$, 4 and 5. For each case both the summer and winter time distributions ($\chi = 0°$ and $\chi = 45°$)are illustrated.

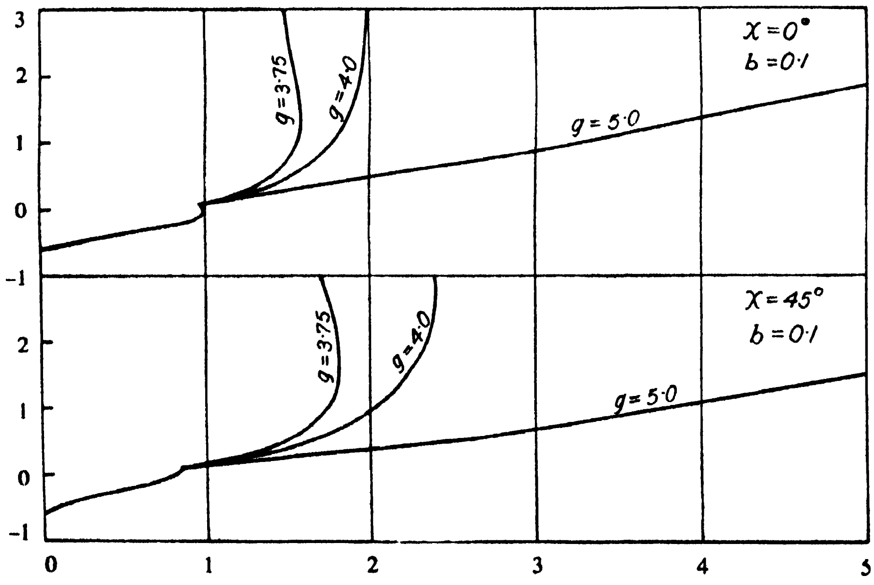

FIG. 9

Altitude distribution of electron concentration for the F-regions on the assumption that $dN/dt = 0$, when the recombination coefficient decreases exponentially and scale height increases linearly with height. Both winter and summer distributions are given and the boundary between the layers is assumed to be about 30 km. above h_0. For each season, three different gradients of recombination coefficient are chosen, namely, g = 3.75, 4.0 and 5.0.

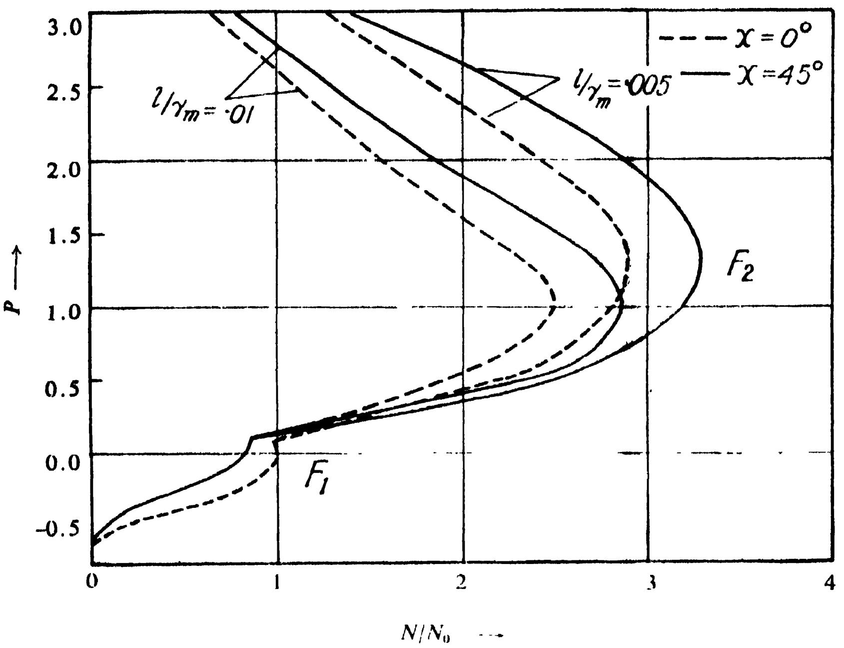

FIG. 10

Approximate distribution of the electron concentration for the F-regions when $g = 5.0$ and $dN/dt \neq 0$. Curves for two different values of (l/γ_m), namely, .01 and .005, and for both $\chi = 0°$ and $\chi = 45°$ are given.

We now apply corrections to take account of the fact that dN/dt is *not* zero. The procedure is similar to that before, the expression for the correction factor C being

$$C = \left[1 + \frac{l}{(1+P)}\right]^{g}$$

The corrected curves are given in figure 10.

It will be observed that the distribution curves for this case are of the same type as those for constant scale height. The main difference is in the ratio (N_{mF2}/N_{mF1}) which, for similar conditions, is greater in this case than in case (i). It is concluded that the effect of a scale height gradient is only to increase the ionization of the upper maximum (i.e., the F_2 layer).

(*c*) *Discussion of the results : The bifurcation phenomen on :*

A glance at the curves in figures 6-10 shows that when the recombination coefficient is decreasing with height, or, when the recombination coefficient is decreasing and the scale height increasing with height, the F_1 layer splits up at the top giving *two* distinct maxima, one at or near the level of maximum ion production and the other higher up. The lower region of ionization is the F_1 region, the upper is the F_2 region. This splitting up of the F_1 region at the top is called *bifurcation*. Let us see how far the results obtained above agree with the observed facts of the bifurcation which are as follows :

(1) At night there is a single F-region. This is true for all localities. With the incidence of solar rays, the upper part of the region begins to separate out. As the solar zenith angle decreases, the lower part (F_1) sinks downwards, while the upper part continues to move up. This results in increasing the separation

that the effect of the above mentioned gradients is least, and the distribution curve approximates to the Chapman layer. For the D region the effects of these gradients are so great that the electron distribution has no resemblance to the Chapman distribution. For the F_1 region the variation of recombination coefficient has this curious effect of splitting it up into two regions F_1 and F_2. The effect of the height variation of scale height is mainly to increase the ionization of the F_2-region. However, for a full explanation of the notoriously anomalous behaviour of the region effects of tidal motions under the influence of the geomagnetic field have to be considered.

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